Trigonometric limits

Trigonometric limit problems revolve around three formulas:

$$\lim_{\theta \to 0} \sin\theta = 0$$

$$\lim_{\theta \to 0} \cos\theta = 1$$

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

When we solve trigonometric limit problems, our goal is always to reduce the function to a combination of nothing but these three formulas and simple constants.

Example

Evaluate the limit.

$$\lim_{x \to 0} \frac{1 - \cos x}{x}$$

Since we have exactly two terms in the numerator, we're actually going to borrow the conjugate method for the first step of this problem.

$$\lim_{x \to 0} \frac{1 - \cos x}{x} \cdot \left(\frac{1 + \cos x}{1 + \cos x} \right)$$

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$$\lim_{x\to 0}\frac{1-\cos^2 x}{x(1+\cos x)}$$

Applying the identity $1-\cos^2 x=\sin^2 x$ to the numerator gives

$$\lim_{x\to 0}\frac{\sin^2 x}{x(1+\cos x)}$$

Notice now that we can factor out $(\sin x)/x$, which is one of our three fundamental formulas.

$$\lim_{x\to 0}\frac{\sin x}{x}\cdot\frac{\sin x}{1+\cos x}$$

$$\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{\sin x}{1+\cos x}$$

Since the first limit is one of our three fundamental formulas, we can replace it with its value.

$$1\cdot\lim_{x\to 0}\frac{\sin x}{1+\cos x}$$

$$\lim_{x\to 0}\frac{\sin x}{1+\cos x}$$

And now we can evaluate the limit at $x\to 0$ without making the denominator 0.

$$\frac{\sin 0}{1+\cos 0}$$

$$\frac{0}{1+1}$$

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$$0$$

So the limit is 0.

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

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